# NEWTON'S METHOD Applied to CURVE DESIGN

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FOR BACKGROUND the reader is referred to the Spring, 1982 edition of The Ontario Land Surveyor containing the article entitled *Computer Solution of Curve Data*. In which, the formula relating β, T, and L (Figure 1) is derived and the advantage of a solution using a computer program to solve for other curve data in terms of tangent length, T, and arc length, L, is explained. The following discussion will present an alternate solution and program utilizing Newton's method.

Newton's method employs the recursive formula $x_i + 1 = x_i - \frac{f(x_i)}{f^1(x_i)}$ where $f^1(x_i)$ represents the first derivative of $f(x)$ evaluated at $x_i$. The iteration is a method of successive substitutions where a value of $x_i + 1$ is calculated from the recursive formula and compared with $x_i$. If the two values are not sufficiently close, then the value for $x_i + 1$ is substituted into the righthand side of the recursive formula and the computation is repeated until the two values are sufficiently close. This is done in lines 50, 60, 70, 120, and 130 of the program (Figure 4).

The relationship between β, T, and L is $\beta - (L \tan\beta)/2T = 0$ (Figure 1). The Newton recursive formula from this relationship is the function of β in Figure 2. The curve will be skewed upward for large T/L ratios, i.e. large central angles, and downward for small. Beginning with an initial estimate of slightly less than 90°, line 50 (Figure 4), the iteration will follow the dashed line (Figure 2). The solution is found where the values of $f(\beta)$ and β (Figure 2) are sufficiently close as determined by line 70 (Figure 4). ●

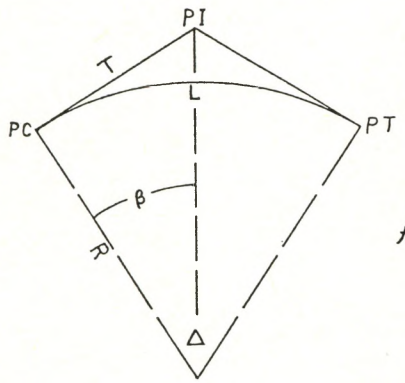


Fig. 1

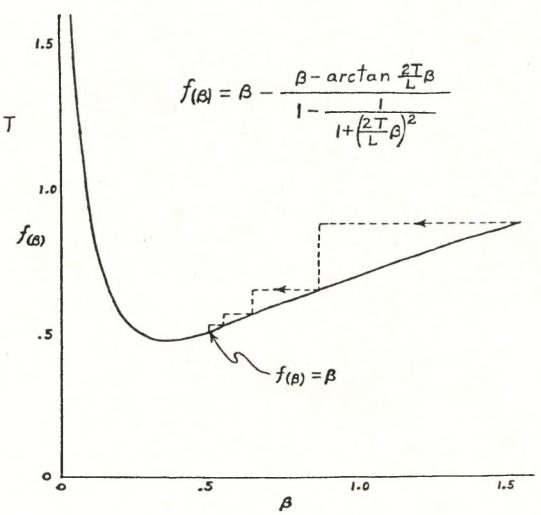


Fig. 2

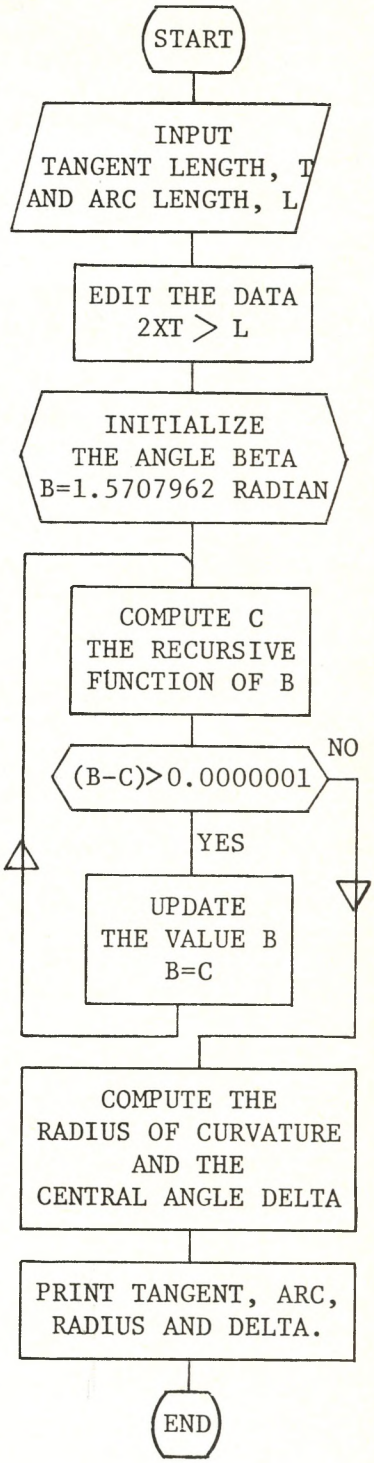


Fig. 3

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10  INPUT T,L
20  IF 2 * T > L THEN 50
30  PRINT "YOU HAVE ENTERED AN IMPOSSIBLE SET OF VALUES FOR T AND L."
40  GOTO 140
50  LET B = 1.5707962
60  LET C = B - (B -  ATN (2 * T * B / L)) / (1 - 1 / (1 + (2 * T * B / L) ^ 2))
70  IF  ABS (B - C) > 0.0000001 THEN 120
80  LET R = T /  TAN (C)
90  LET D = 360 * C / 3.1415927
100  PRINT "FOR TANGENT LENGTH "T" AND ARC LENGTH "L" THE RADIUS IS "R
101  PRINT "AND THE CENTRAL ANGLE, DELTA, IS "D" DEGREES"
110  GOTO 140
120  LET B = C
130  GOTO 60
140  END
```

Fig. 4

TEST RESULTS

| TAN | ARC | RADIUS | DELTA |
|---|---|---|---|
| 107.15 | 234.79 | WON'T COMPUTE | |
| 100.01 | 200.00 | 5773.83259 | 1.98467059 |
| 101.02 | 200.02 | 578.042632 | 19.8260491 |
| 112.37 | 197.63 | 166.251945 | 68.1096673 |
| 227.81 | 211.37 | 87.8779827 | 137.811636 |
| 203.05 | 97.43 | 34.7660994 | 160.568126 |
| 947.05 | 3.72 | 1.18505678 | 179.856607 |

Fig. 5